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10 & 10A

ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM
THIRD EDITION

DAVID GREENWOOD
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ISBN 978-1-108-77346-1

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108773461

First published 2011
Second Edition 2015
Third Edition 2020
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover and text designed by Loupe Studio
Typeset by diacriTech
Printed in China by C & C Offset Printing Co. Ltd.

A catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-108-77346-1 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

Contents

ISBN 978-1-108-77346-1

ISBN 978-1-108-77346-1

ISBN 978-1-108-77346-1

ISBN 978-1-108-77346-1

ISBN 978-1-108-77346-1

About the authors

David Greenwood is the Head of Mathematics at Trinity Grammar School, Melbourne and has 21 years' experience teaching Year 12 mathematics. He has run numerous workshops within Australia on the implementation of the Australian Curriculum for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006, has written more than 10 mathematics titles and specialises in lesson design and differentiation.

Jenny Goodman has worked for 20 years in comprehensive state and selective high schools in New South Wales and has a keen interest in teaching students of differing ability levels. She was awarded the Jones Medal for Education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

Stuart Palmer has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over New South Wales and beyond. He is a Project Officer with the Mathematical Association of New South Wales, and also works with pre-service teachers at The University of Sydney and Western Sydney University.

ISBN 978-1-108-77346-1

Introduction

This third edition of *Essential Mathematics for the Australian Curriculum* includes some substantial new features in the print and digital versions of the textbook, as well as in the Online Teaching Suite. The main new features are listed below.

Now you try

Every worked example now contains additional questions, without solutions, called ‘Now you try’. Rather than expect students to absorb the worked example by passively reading through it, these questions give students immediate practice at the same type of question. We also anticipate these questions will be useful for the teacher to do in front of the class, given that students will not have seen the solution beforehand.

Building understanding and changes to the exercise structure

To improve the flow of ideas from the beginning of each lesson through to the end of the exercise, a few structural changes have been made in each lesson. First, the Understanding questions have been taken out of the exercise, simplified into discussion-style questions, and placed immediately after the Key ideas. These questions are now called ‘Building understanding’ and are intended to consolidate the skills and concepts covered by the Key ideas, which students will then encounter in the worked examples. Each exercise now starts at Fluency, and the first question in each exercise has been revised to ensure that it links directly to the first worked example in the lesson. The exercise then continues as before through Problem-solving, Reasoning and Enrichment.

Learning intentions and Success criteria checklist

At the beginning of every lesson is a set of Learning intentions that describe what the student can expect to learn in the lesson. At the end of the chapter, these appear again in the form of a Success criteria checklist; students can use this to check their progress through the chapter. Every criterion is listed with an example question to remind students of what the mathematics mentioned looks like. These checklists can also be downloaded and printed off so that students can physically check them off as they accomplish their goals.

Modelling and more extended-response

A modelling activity now accompanies the Investigation in each chapter, with the goal of familiarising students with using the modelling process to define, solve, verify and then communicate their solutions to real-life problems. Also included in each chapter is a set of three applications and problem-solving questions. These extended-response style problems apply the mathematics of the chapter to realistic contexts and provide important practice at this type of extended-response work before any final test is completed.

Workspaces and self-assessment

In the Interactive Textbook, students can complete almost any question from the textbook inside the platform via **workspaces**. Questions can be answered with full worked solutions using three input tools: ‘handwriting’ using a stylus, inputting text via a keyboard and in-built symbol palette, or uploading an image of work completed elsewhere. Then students can critically engage with their own work using the **self-assessment** tools, which allow them to rate their confidence with their work and also red-flag to the teacher any questions they have not understood. All work is saved, and teachers will be able to see both students’ working-out and how they’ve assessed their own work via the Online Teaching Suite.

Note that the workspaces and self-assessment feature is intended to be used as much or as little as the teacher wishes, including not at all. However, the ease with which useful data can be collected will make this feature a powerful teaching and learning tool when used creatively and strategically.

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Guide to the working programs

As with the second edition, *Essential Mathematics for the Australian Curriculum Third Edition* contains working programs that are subtly embedded in every exercise. The suggested working programs provide three pathways through each book to allow differentiation for Foundation, Standard and Advanced students.

Each exercise is structured in subsections that match the Australian Curriculum proficiency strands of Fluency, Problem-solving and Reasoning, as well as Enrichment (Challenge). (Note that Understanding is now covered by 'Building understanding' in each lesson.) In the exercises, the questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest shaded colour) is the Foundation pathway
- The middle column (medium shaded colour) is the Standard pathway
- The right column (darkest shaded colour) is the Advanced pathway.

	Foundation	Standard	Advanced
FLUENCY	1, 2–4(½)	2–5(½)	2–5(½)
PROBLEM-SOLVING	6, 7	6–8	7–9
REASONING	10	10–12	12–14
ENRICHMENT	–	–	15

The working program for Exercise 3A in Year 7

Gradients within exercises and proficiency strands

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Fluency through to Reasoning and Enrichment – but also within each proficiency strand; the first few questions in Fluency, for example, are easier than the last few, and the last Problem-solving question is more challenging than the first Problem-solving question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Foundation pathway should use the left tab, which includes all but the hardest Fluency questions as well as the easiest Problem-solving and Reasoning questions. An Advanced student can use the right tab, proceed through the Fluency questions (often half of each question), with their main focus on the Problem-solving and Reasoning questions, as well as the Enrichment questions. A Standard student would do a mix of everything using the middle tab.

Choosing a pathway

There are a variety of ways to determine the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them. If required, there are two types of chapter pre-tests (now found online) that can be used as a tool for helping students select a pathway. For the prior-knowledge pre-test, the following are recommended guidelines:

- A student who gets 40% or lower should complete the Foundation questions
- A student who gets between 40% and 85% should complete the Standard questions
- A student who gets 85% or higher should complete the Advanced questions.

For schools that have classes grouped according to ability, teachers may wish to set one of the Foundation, Standard or Advanced pathways as their default setting for their entire class and then make individual alterations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : do not complete any of the questions in this section.

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Guide to this resource

PRINT TEXTBOOK FEATURES

1 **Australian Curriculum:** content strands, sub-strands and content descriptions are listed at the beginning of the chapter (see the teaching program for more detailed curriculum documents)

2 **Working with unfamiliar problems:** a set of problem-solving questions not tied to a specific topic

3 NEW **Learning intentions:** sets out what a student will be expected to learn in the lesson

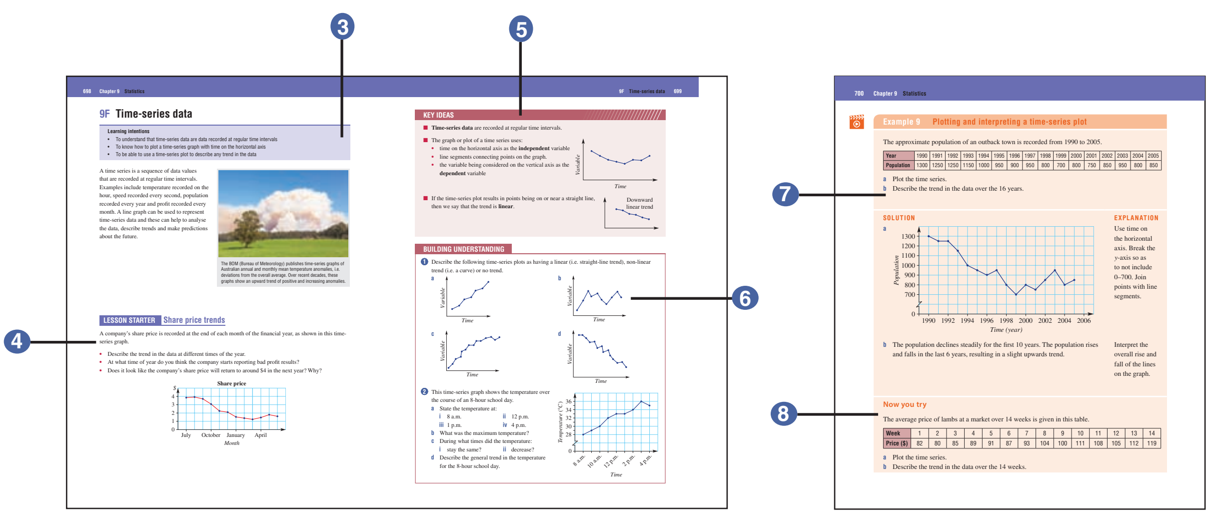
4 **Lesson starter:** an activity, which can often be done in groups, to start the lesson

5 **Key ideas:** summarises the knowledge and skills for the lesson

6 NEW **Building understanding:** a small set of discussion questions to consolidate understanding of the Key ideas (replaces Understanding questions formerly inside the exercises)

7 **Worked examples:** solutions and explanations of each line of working, along with a description that clearly describes the mathematics covered by the example

8 NEW **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give immediate practice



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9 Revised exercise structure: the exercise now begins at Fluency, with the first question always linked to the first worked example in the lesson.

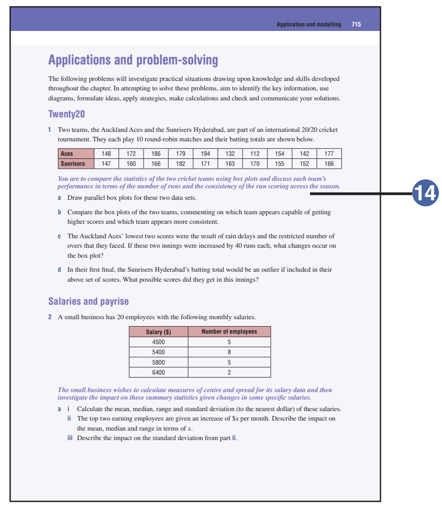
10 Working programs: differentiated question sets for three ability levels in exercises

11 Example references: show where a question links to a relevant worked example – the first question is always linked to the first worked example in a lesson

12 NEW Non-CAS TI and Casio calculator activities added for Years 9 (online) and 10 & 10A (print)

13 NEW Modelling activities in every chapter allow students to practise solving problems using a systematic modelling process

14 NEW Applications and problem-solving: a set of three extended-response questions across two pages that give practice at applying the mathematics of the chapter to real-life contexts

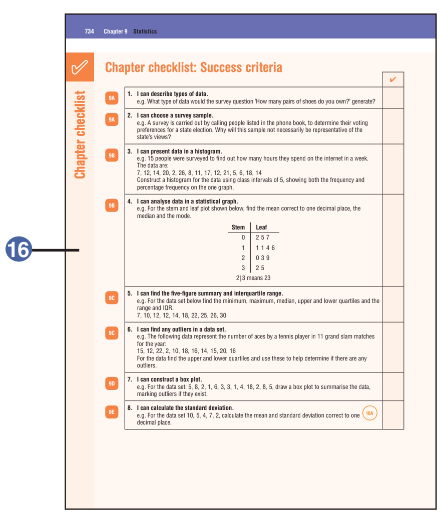


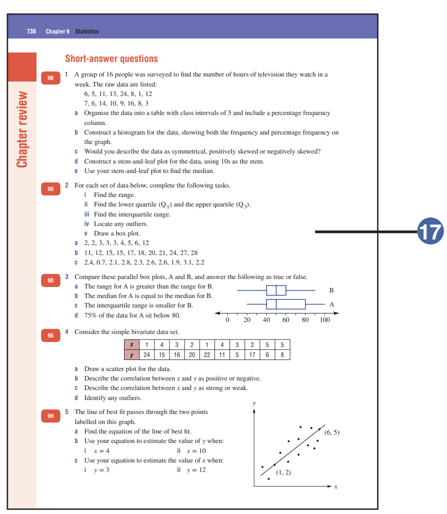
15 Problems and challenges: in each chapter provide practice with solving problems connected with the topic

16 NEW Success criteria checklist: a checklist of the learning intentions for the chapter, with example questions

17 Chapter reviews: with short-answer, multiple-choice and extended-response questions; questions that are extension or 10A (at Year 10) are clearly signposted

18 Solving unfamiliar problems poster: at the back of the book outlines a strategy for solving any unfamiliar problem





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INTERACTIVE TEXTBOOK FEATURES

19 NEW **Workspaces:** almost every textbook question – including all working-out – can be completed inside the Interactive Textbook by using either a stylus, a keyboard and symbol palette, or uploading an image of the work.

20 NEW **Self-assessment:** students can then self-assess their own work and send alerts to the teacher. See the Introduction on page ix for more information

21 **Interactive question tabs** can be clicked on so that only questions included in that working program are shown on the screen

22 **HOTmaths resources:** a huge catered library of widgets, HOTsheets and walkthroughs seamlessly blended with the digital textbook

23 A revised set of **differentiated auto-marked practice quizzes** per lesson with saved scores

24 **Scorcher:** the popular competitive game

25 **Worked example videos:** every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the flipped classroom

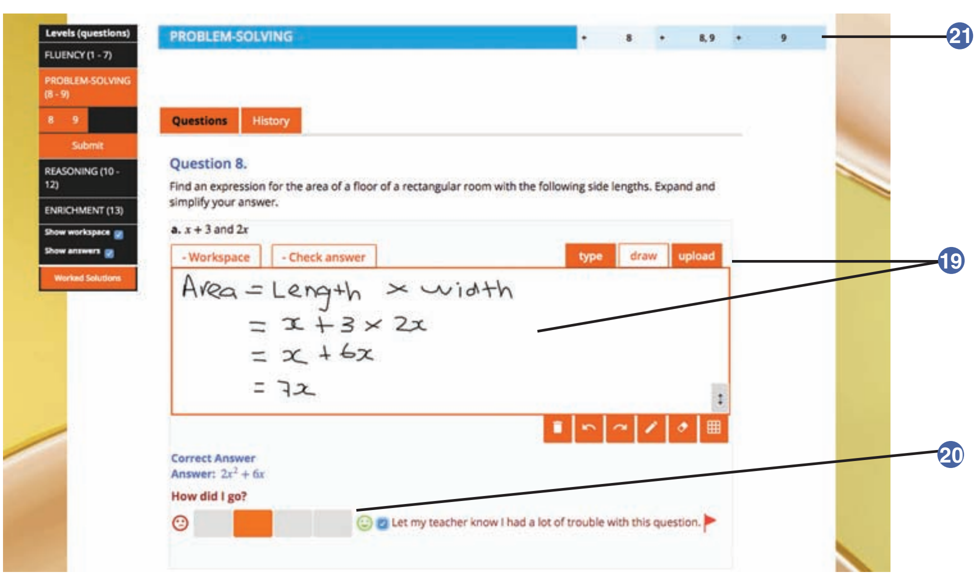
26 **Desmos graphing calculator**, scientific calculator and geometry tool are always available to open within every lesson

27 **Desmos interactives:** a set of Desmos activities written by the authors allow students to explore a key mathematical concept by using the Desmos graphing calculator or geometry tool

28 **Auto-marked prior knowledge pre-test** for testing the knowledge that students will need before starting the chapter

29 NEW **Auto-marked diagnostic pre-test** for setting a baseline of knowledge of chapter content

30 **Auto-marked progress quizzes and chapter review multiple-choice questions** in the chapter reviews can now be completed online



ISBN 978-1-108-77346-1

DOWNLOADABLE PDF TEXTBOOK

31 In addition to the Interactive Textbook, a **PDF version of the textbook** has been retained for times when users cannot go online. PDF search and commenting tools are enabled.

ONLINE TEACHING SUITE

32 **Learning Management System** with class and student analytics, including reports and communication tools

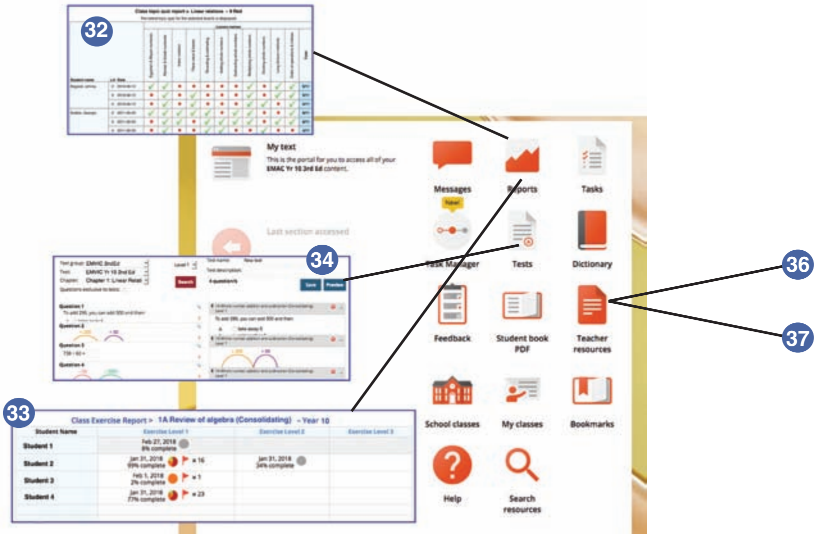
33 NEW **Teacher view of student's work and self-assessment** allows the teacher to see their class's workout, how students in the class assessed their own work, and any 'red flags' that the class has submitted to the teacher

34 **Powerful test generator** with a huge bank of levelled questions as well as ready-made tests

35 NEW **Revamped task manager** allows teachers to incorporate many of the activities and tools listed above into teacher-controlled learning pathways that can be built for individual students, groups of students and whole classes.

36 **Worksheets and four differentiated chapter tests in every chapter**, provided in editable Microsoft Word documents

37 NEW **More printable resources:** all Pre-tests, Progress quizzes and Applications and problem-solving tasks are provided in printable worksheet versions



ISBN 978-1-108-77346-1

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The author and publisher wish to thank the following sources for permission to reproduce material:

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Part 1

Working with unfamiliar problems: Part 1

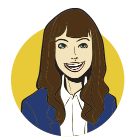
The questions on the next four pages are designed to provide practice in solving unfamiliar problems. Use the 'Working with unfamiliar problems' poster at the back of this book to help you if you get stuck.

In Part 1, apply the suggested strategy to solve these problems, which are in no particular order. Clearly communicate your solution and final answer.

1 Discover the link between Pascal's triangle and expanded binomial products and use this pattern to help you expand $(x + y)^6$.

Pascal's triangle

$(x+y)^0$				1			
$(x+y)^1$			1		1		
$(x+y)^2$		1		2		1	
$(x+y)^3$	1		3		3		1



For Question 1, try looking for number patterns and algebraic patterns.

2 How many palindromic numbers are there between 10^1 and 10^3?

3 Find the smallest positive integer values for x so that $60x$ is:

a a perfect square

b a perfect cube

c divisible by both 8 and 9.

For Questions 2 and 3, try making a list or table.

4 A Year 10 class raises money at a fete by charging players \$1 to flip their dollar coin onto a red and white checked tablecloth with 50 mm squares. If the dollar coin lands fully inside a red square the player keeps their \$1. What is the probability of keeping the \$1?
How much cash is likely to be raised from 64 players?

5 The shortest side of a 60° set square is 12 cm. What is the length of the longest side of this set square?

6 A Ferris wheel with diameter 24 metres rotates at a constant rate of 60 seconds per revolution.

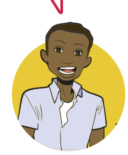
a Calculate the time taken for a rider to travel:

i from the bottom of the wheel to 8 m vertically above the bottom

ii from 8 m to 16 m vertically above the bottom of the wheel.

b What fraction of the diameter is the vertical height increase after each one-third of the ride from the bottom to the top of the Ferris wheel?

For Questions 4–8, try drawing a diagram to help you visualise the problem.



7 $ABCD$ is a rectangle with $AB = 16$ cm and $AD = 12$ cm. X and Y are points on BD such that AX and CY are each perpendicular to the diagonal BD. Find the length of the interval XY.

8 How many diagonal lines can be drawn inside a decagon (i.e. a 10-sided polygon)?

ISBN 978-1-108-77346-1

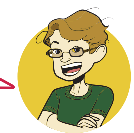
Cambridge University Press

9 The symbol ! means factorial.
e.g. $4! = 4 \times 3 \times 2 \times 1 = 24$.
Simplify $9! \div 7!$ without the use of a calculator.

For Question 9, try to break up the numbers to help simplify.

10 In 2017 Charlie's age is the sum of the digits of his birth year $19xy$ and Bob's age is one less than triple the sum of the digits of his birth year $19yx$. Find Charlie's age and Bob's age on their birthdays in 2017.

For Question 10, try to set up an equation

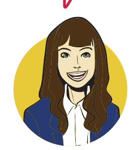


11 Let D be the difference between the squares of two consecutive positive integers. Find an expression for the average of the two integers in terms of D.

12 For what value of b is the expression $15ab + 6b - 20a - 8$ equal to zero for all values of a?

13 Find the value of k given $k > 0$ and that the area enclosed by the lines $y = x + 3, x + y + 5 = 0, x = k$ and the y-axis is 209 units^2.

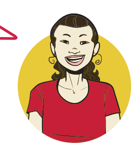
For Questions 11–13, try using algebra as a tool to work out the unknowns.



14 The diagonal of a cube is $\sqrt{27}$ cm. Calculate the volume and surface area of this cube.

15 Two sides of a triangle have lengths 8 cm and 12 cm, respectively. Determine between which two values the length of the third side would fall. Give reasons for your answer.

For Questions 14 and 15, try using concrete, everyday materials to help you understand the problem.

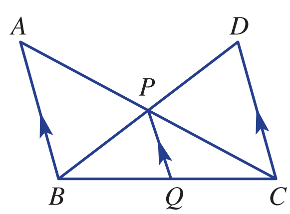


16 When $10^{89} - 89$ is expressed as a single number, what is the sum of its digits?

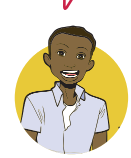
17 Determine the reciprocal of this product: $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{n+1}\right)$.

18 Find the value of $\dfrac{1002^2 - 998^2}{102^2 - 98^2}$, without using a calculator.

19 In the diagram at right, $AP = 9$ cm, $PC = 15$ cm, $BQ = 8.4$ cm and $QC = 14$ cm. Also, $CD \parallel QP \parallel BA$. Determine the ratio of the sides AB to DC.



For Questions 16–19, try using a mathematical procedure to find a shortcut to the answer.



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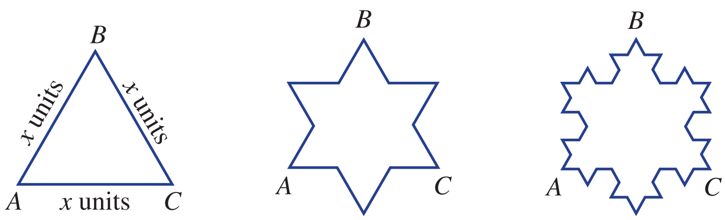
Working with unfamiliar problems: Part 2

For the questions in Part 2, again use the 'Working with unfamiliar problems' poster at the back of this book, but this time choose your own strategy (or strategies) to solve each problem. Clearly communicate your solution and final answer.

1 The Koch snowflake design starts with an equilateral triangle. A smaller equilateral triangle is built onto the middle third of each side and its base is erased. This procedure can be repeated indefinitely.



a For a Koch snowflake with initial triangle side length x units, determine expressions for the exact value of:

i the perimeter after 5 procedure repeats and after n procedure repeats

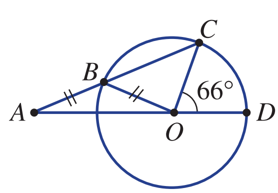
ii the sum of areas after 3 procedure repeats and the *change* in area after n procedure repeats.

b Comment on perimeter and area values as $n \to \infty$. Give reasons for your answers.

2 Two sides of a triangle have lengths in the ratio 3 : 5 and the third side has length 37 cm. If each side length has an integer value, find the smallest and largest possible perimeters, in cm.

3 The midpoints of each side of a regular hexagon are joined to form a smaller regular hexagon with side length k cm. Determine a simplified expression in terms of k for the exact difference in the perimeters of the two hexagons.

4 Angle COD is 66°. Find the size of angle CAD.



5 The graph of $y = ax^2 + 2x + 3$ has an axis of symmetry at $x = \frac{1}{4}$. Determine the maximum possible value of y.

6 Find the value of x and y given that $5^x = 125^{y-3}$ and $81^{x+1} = 9^y \times 3$.

7 A rectangular prism has a surface area of 96 cm^2 and the sum of the lengths of all its edges is 64 cm. Determine the exact sum of the lengths of all its internal diagonals (i.e. diagonals not on a face).

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8 In a Year 10 maths test, six students gained 100%, all students scored at least 75% and the mean mark was 82.85%. If the results were all whole numbers, what is the smallest possible number of students in this class? List the set of results for this class size.

9 Determine the exact maximum vertical height of the line $y = 2x$ above the parabola $y = 2x^2 - 5x - 3$.

10 $A + B = 6$ and $AB = 4$. Without solving for A and B, determine the values of:

a $(A + 1)(B + 1)$ **b** $A^2 + B^2$ **c** $(A - B)^2$ **d** $\frac{1}{A} + \frac{1}{B}$

11 If $f(1) = 5$ and $f(x + 1) = 2f(x)$, determine the value of $f(8)$.

12 Four rogaining markers, $PQRS$, are in an area of bushland with level ground. Q is 1.4 km east of P, S is 1 km from P on a true bearing of 168° and R is 1.4 km from Q on a true bearing of 200°. To avoid swamps, Lucas runs the route $PRSQP$. Calculate the distances (in metres) and the true bearings from P to R, from R to S, from S to Q and from Q to P. Round your answers to the nearest whole number.

13 Consider all points (x, y) that are equidistant from the point $(4, 1)$ and the line $y = -3$. Find the rule relating x and y and then sketch its graph, labelling all significant features. (Note: Use the distance formula.)

14 A 'rule of thumb' useful for 4WD beach driving is that the proportion of total tide height change after either high or low tide is $\frac{1}{12}$ in the first hour, $\frac{2}{12}$ in the second hour, $\frac{3}{12}$ in the third hour, $\frac{3}{12}$ in the fourth hour, $\frac{2}{12}$ in the fifth hour and $\frac{1}{12}$ in the sixth hour.

a Determine the accuracy of this 'rule of thumb' using the following equation for tide height: $h = 0.7 \cos(30t) + 1$, where h is in metres and t is time in hours after high tide.

b Using $h = A \cos(30t) + D$, show that the proportion of total tide height change between any two given times, t_1 and t_2, is independent of the values of A and D.

15 All Golden Rectangles have the proportion $L : W = \Phi : 1$ where Φ (phi) is the Golden Number. Every Golden Rectangle can be subdivided into a square of side W and a smaller Golden Rectangle. Calculate phi as an exact number and also to six decimal places.

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CHAPTER 1 Linear relations

Rally driving and suspension

Rally drivers need to rely on their car's suspension to keep them safe on difficult and intense tracks around the world. Drivers need to understand how their suspension system works in order to ensure that it performs as it should under extreme conditions.

The springs used in a suspension system obey the principles of a linear relationship. The force acting on the springs versus the extension of the springs forms a linear relation. Mechanics investigate the performance of various suspension springs that are suitable for the make and model of the rally car and decide which is the best for the car and driver.

Linear relationships in cars are not just restricted to suspension systems; running costs and speed efficiency can also be modelled using linear relationships. Fixed costs are the y-intercept of the equation and then the variable costs are set as the gradient, as this gives the rate of increase in the overall costs per kilometre.

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

Australian Curriculum

NUMBER AND ALGEBRA
Patterns and algebra

Apply the four operations to simple algebraic fractions with numerical denominators. (ACMNA232)

Substitute values into formulas to determine an unknown. (ACMNA234)

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas. (ACMNA235)

Solve linear inequalities and graph their solutions on a number line. (ACMNA236)

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology. (ACMNA237)

Solve problems involving parallel and perpendicular lines. (ACMNA238)

Solve linear equations involving simple algebraic fractions. (ACMNA240)

Essential Mathematics for the Australian Curriculum Year 10 & 10A 3ed
ISBN 978-1-108-77346-1

1A Review of algebra CONSOLIDATING

Learning intentions

- To review the key words of algebra: term, coefficient, expression, equation
- To review how to combine like terms under addition and subtraction
- To review how to multiply and divide algebraic terms and apply the distributive law to expand brackets
- To review how to factorise an expression using the highest common factor
- To be able to substitute values for pronumerals and evaluate expressions

Algebra involves the use of pronumerals (or variables), which are letters representing numbers. Combinations of numbers and pronumerals form terms (numbers and pronumerals connected by multiplication and division), expressions (a term or terms connected by addition and subtraction) and equations (mathematical statements that include an equals sign). Skills in algebra are important when dealing with the precise and concise nature of mathematics. The complex nature of many problems in finance and engineering usually result in algebraic expressions and equations that need to be simplified and solved.

Stockmarket traders rely on financial modelling based on complex algebraic expressions. Financial market analysts and computer systems analysts require advanced algebraic skills.

LESSON STARTER Mystery problem

Between one school day and the next, the following problem appeared on a student noticeboard.

Prove that $8 - x^2 + \dfrac{3x - 9}{3} + 5(x - 1) - x(6 - x) = 0$.

- By working with the left-hand side of the equation, show that this equation is true for any value of x.
- At each step of your working, discuss what algebraic processes you have used.

KEY IDEAS

- Key words in algebra:
  - **term:** $5x$, $7x^2y$, $\dfrac{2a}{3}$, 7 (a constant term)
  - **coefficient:** -3 is the coefficient of x^2 in $7 - 3x^2$; 1 is the coefficient of y in $y + 7x$.
  - **expression:** $7x$, $3x + 2xy$, $\dfrac{x + 3}{2}$, $\sqrt{2a^2 - b}$
  - **equation:** $x = 5$, $7x - 1 = 2$, $x^2 + 2x = -4$
- Expressions can be evaluated by substituting a value for each pronumeral (variable).
  - Order of operations are followed: First brackets, then indices, then multiplication and division, then addition and subtraction, working then from left to right.

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- **Like terms** have the same pronumeral part and, using addition and subtraction, can be collected to form a single term.
  For example, $3x - 7x + x = -3x$
  $6a^2b - ba^2 = 5a^2b$
  Note that $a^2b = ba^2$
- The symbols for multiplication (×) and division (÷) are usually not shown.
  $7 \times x \div y = \dfrac{7x}{y}$
  $-6a^2b \div (ab) = \dfrac{-6a^2b}{ab}$
  $= -6a$
- The **distributive law** is used to expand brackets.
  - $a(b + c) = ab + ac$ $\quad 2(x + 7) = 2x + 14$
  - $a(b - c) = ab - ac$ $\quad -x(3 - x) = -3x + x^2$
- **Factorisation** involves writing expressions as a product of factors.
  - Many expressions can be factorised by taking out the highest common factor (HCF).
    $15 = 3 \times 5$
    $3x - 12 = 3(x - 4)$
    $9x^2y - 6xy + 3x = 3x(3xy - 2y + 1)$
- Other general properties are:
  - **associative** $a \times (b \times c) = (a \times b) \times c$ or $a + (b + c) = (a + b) + c$
  - **commutative** $ab = ba$ or $a + b = b + a$ $\left(\text{Note: } \dfrac{a}{b} \neq \dfrac{b}{a} \text{ and } a - b \neq b - a.\right)$
  - **identity** $a \times 1 = a$ or $a + 0 = a$
  - **inverse** $a \times \dfrac{1}{a} = 1$ or $a + (-a) = 0$

BUILDING UNDERSTANDING

1. Which of the following is an equation?
   A $3x - 1$ **B** $\dfrac{x + 1}{4}$ **C** $7x + 2 = 5$ **D** $3x^2y$
2. Which expression contains a term with a coefficient of -9?
   A $8 + 9x$ **B** $2x + 9x^2y$ **C** $9a - 2ab$ **D** $b - 9a^2$
3. State the coefficient of a^2 in these expressions.
   a $a + a^2$ **b** $\dfrac{3}{2} - 4a^2$ **c** $1 - \dfrac{a^2}{5}$ **d** $-\dfrac{7a^2}{3} - 1$
4. Decide whether the following pairs of terms are like terms.
   a xy and $2yx$ **b** $7a^2b$ and $-7ba^2$ **c** $-4abc^2$ and $8ab^2c$
5. Evaluate:
   a $(-3)^2$ **b** $(-2)^3$ **c** -2^3 **d** -3^2

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Example 1 Collecting like terms

Simplify by collecting like terms.

a $7a + 3a$ **b** $3a^2b - 2a^2b$ **c** $5xy + 2xy^2 - 2xy + 3y^2x$

SOLUTION

a $7a + 3a = 10a$

b $3a^2b - 2a^2b = a^2b$

c $5xy + 2xy^2 - 2xy + 3y^2x = 3xy + 5xy^2$

EXPLANATION

Keep the pronumeral and add the coefficients.

$3a^2b$ and $2a^2b$ have the same pronumeral part, so they are like terms. Subtract coefficients and recall that $1a^2b = a^2b$.

Collect like terms, noting that $3y^2x = 3xy^2$. The $+$ or $-$ sign belongs to the term that directly follows it.

Now you try

Simplify by collecting like terms.

a $4a + 13a$ **b** $5ab^2 - 2ab^2$ **c** $3xy + 4x^2y - xy + 2yx^2$

Example 2 Multiplying and dividing expressions

Simplify the following.

a $2h \times 7l$ **b** $-3p^2r \times 2pr$ **c** $-\frac{7xy}{14y}$

SOLUTION

a $2h \times 7l = 14hl$

b $-3p^2r \times 2pr = -6p^3r^2$

c $-\frac{7xy}{14y} = -\frac{x}{2}$

EXPLANATION

Multiply the coefficients and remove the $\times$ sign.

Remember the basic index law: When you multiply terms with the same base you add the powers.

Cancel the highest common factor of 7 and 14 and cancel the y.

Now you try

Simplify the following.

a $3a \times 6b$ **b** $-2x^2y \times 5xy$ **c** $-\frac{4ab}{8a}$

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Example 3 Expanding the brackets

Expand the following using the distributive law. Simplify where possible.

a $2(x + 4)$ **b** $-3x(x - y)$ **c** $3(x + 2) - 4(2x - 4)$

SOLUTION

a $2(x + 4) = 2x + 8$

b $-3x(x - y) = -3x^2 + 3xy$

c $3(x + 2) - 4(2x - 4) = 3x + 6 - 8x + 16$
$= -5x + 22$

EXPLANATION

$2(x + 4) = 2 \times x + 2 \times 4$

Note that $x \times x = x^2$ and $-3 \times (-1) = 3$.

Expand each pair of brackets and simplify by collecting like terms.

Now you try

Expand the following using the distributive law. Simplify where possible.

a $3(x + 2)$ **b** $-2x(x - y)$ **c** $2(x + 3) - 3(2x - 1)$

Example 4 Factorising simple algebraic expressions

Factorise:

a $3x - 9$ **b** $2x^2 + 4x$

SOLUTION

a $3x - 9 = 3(x - 3)$

b $2x^2 + 4x = 2x(x + 2)$

EXPLANATION

HCF of $3x$ and 9 is 3.

Check that $3(x - 3) = 3x - 9$.

HCF of $2x^2$ and $4x$ is $2x$.

Check that $2x(x + 2) = 2x^2 + 4x$.

Now you try

Factorise:

a $2x - 10$ **b** $3x^2 + 9x$

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Example 5 Evaluating expressions

Evaluate $a^2 - 2bc$ if $a = -3$, $b = 5$ and $c = -1$.

SOLUTION	EXPLANATION
$a^2 - 2bc = (-3)^2 - 2(5)(-1)$	Substitute for each pronumeral:
$= 9 - (-10)$	$(-3)^2 = -3 \times (-3)$ and $2 \times 5 \times (-1) = -10$
$= 19$	To subtract a negative number, add its opposite.

Now you try

Evaluate $b^2 - 3ac$ if $a = 1$, $b = -2$ and $c = -3$.

Exercise 1A

FLUENCY	1, 2–7(1/2)	2–7(1/2)	2–7(1/3)

1 Simplify by collecting like terms.

Example 1a — **a** **i** $5a + 9a$ **ii** $7a - 2a$

Example 1b — **b** **i** $4a^2b - 2a^2b$ **ii** $5x^2y - 4x^2y$

Example 1c — **c** **i** $4xy + 3xy^2 - 3xy + 2y^2x$ **ii** $6ab + 2ab^2 - 2ab + 4b^2a$

Example 1

2 Simplify by collecting like terms.

a $6a + 4a$
b $8d + 7d$
c $5y - 5y$
d $2xy + 3xy$
e $9ab - 5ab$
f $4t + 3t + 2t$
g $7b - b + 3b$
h $3st^2 - 4st^2$
i $4m^2n - 7nm^2$
j $0.3a^2b - ba^2$
k $4gh + 5 - 2gh$
l $7xy + 5xy - 3y$
m $4a + 5b - a + 2b$
n $3jk - 4j + 5jk - 3j$
o $2ab^2 + 5a^2b - ab^2 + 5ba^2$
p $3mn - 7m^2n + 6nm^2 - mn$
q $4st + 3ts^2 + st - 4s^2t$
r $7x^3y^4 - 3xy^2 - 4y^4x^3 + 5y^2x$

Example 2

3 Simplify the following.

a $4a \times 3b$
b $5a \times 5b$
c $-2a \times 3d$
d $5h \times (-2m)$
e $-6h \times (-5t)$
f $-5b \times (-6l)$
g $2s^2 \times 6t$
h $-3b^2 \times 7d^5$
i $4ab \times 2ab^3$
j $-6p^2 \times (-4pq)$
k $6hi^4 \times (-3h^4i)$
l $7mp \times 9mr$
m $\frac{7x}{7}$
n $\frac{6ab}{2}$
o $-\frac{3a}{9}$
p $-\frac{2ab}{8}$
q $\frac{4ab}{2a}$
r $-\frac{15xy}{5y}$
s $-\frac{4xy}{8x}$
t $-\frac{28ab}{56b}$

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Example 3a,b

4 Expand the following, using the distributive law.

a $5(x + 1)$ **b** $2(x + 4)$ **c** $3(x - 5)$

d $-5(4 + b)$ **e** $-2(y - 3)$ **f** $-7(a + c)$

g $-6(-m - 3)$ **h** $4(m - 3n + 5)$ **i** $-2(p - 3q - 2)$

j $2x(x + 5)$ **k** $6a(a - 4)$ **l** $-4x(3x - 4y)$

m $3y(5y + z - 8)$ **n** $9g(4 - 2g - 5h)$ **o** $-2a(4b - 7a + 10)$

p $7y(2y - 2y^2 - 4)$ **q** $-3a(2a^2 - a - 1)$ **r** $-t(5t^3 + 6t^2 + 2)$

s $2m(3m^3 - m^2 + 5m)$ **t** $-x(1 - x^3)$ **u** $-3s(2t - s^3)$

Example 3c

5 Expand and simplify the following, using the distributive law.

a $2(x + 4) + 3(x + 5)$ **b** $4(a + 2) + 6(a + 3)$

c $6(3y + 2) + 3(y - 3)$ **d** $3(2m + 3) + 3(3m - 1)$

e $2(2 + 6b) - 3(4b - 2)$ **f** $3(2t + 3) - 5(2 - t)$

g $2x(x + 4) + x(x + 7)$ **h** $4(6z - 4) - 3(3z - 3)$

i $3d^2(2d^3 - d) - 2d(3d^4 + 4d^2)$ **j** $q^3(2q - 5) + q^2(7q^2 - 4q)$

Example 4

6 Factorise:

a $3x - 9$ **b** $4x - 8$ **c** $10y + 20$

d $6y + 30$ **e** $x^2 + 7x$ **f** $2a^2 + 8a$

g $5x^2 - 5x$ **h** $9y^2 - 63y$ **i** $xy - xy^2$

j $x^2y - 4x^2y^2$ **k** $8a^2b + 40a^2$ **l** $7a^2b + ab$

m $-5t^2 - 5t$ **n** $-6mn - 18mn^2$ **o** $-y^2 - 8yz$

Example 5

7 Evaluate these expressions if $a = -4$, $b = 3$ and $c = -5$.

a $-2a^2$ **b** $b - a$

c $abc + 1$ **d** $-ab$

e $\dfrac{a + b}{2}$ **f** $\dfrac{3b - a}{5}$

g $\dfrac{a^2 - b^2}{c}$ **h** $\dfrac{\sqrt{a^2 + b^2}}{\sqrt{c^2}}$

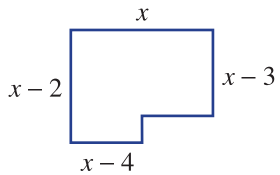
PROBLEM-SOLVING	8	8, 9	9

8 Find an expression for the area of a floor of a rectangular room with the following side lengths. Expand and simplify your answer.

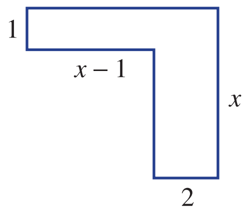
a $x + 3$ and $2x$ **b** x and $x - 5$

9 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. (Note: All angles are right angles.)

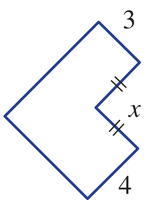
a



b



c



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REASONING	10	10, 11	11, 12

10 When $a = -2$ give reasons why:

a $a^2 > 0$

b $-a^2 < 0$

c $a^3 < 0$

11 Decide whether the following are true or false for all values of a and b. If false, give an example to show that it is false.

a $a + b = b + a$

b $a - b = b - a$

c $ab = ba$

d $\frac{a}{b} = \frac{b}{a}$

e $a + (b + c) = (a + b) + c$

f $a - (b - c) = (a - b) - c$

g $a \times (b \times c) = (a \times b) \times c$

h $a \div (b \div c) = (a \div b) \div c$

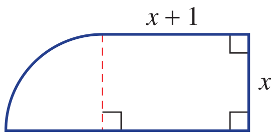
12 a Write an expression for the statement 'the sum of x and y divided by 2'.

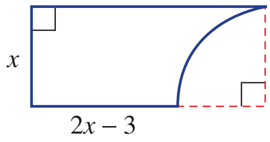
b Explain why the statement above is ambiguous.

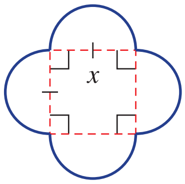
c Write an unambiguous statement describing $\frac{a + b}{2}$.

ENRICHMENT: Algebraic circular spaces	–	–	13

13 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. Your answers may contain π, for example 4π. Do not use decimals.

a


b


c


Architecture, building, carpentry and landscaping are among the many occupations that use algebraic formulas to calculate areas and perimeters in daily work.

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1B Multiplying and dividing algebraic fractions

Learning intentions

- To understand that expressions in algebraic fractions need to be in factorised form in order to cancel common factors
- To know that it is helpful to cancel common factors in fractions before multiplying or dividing
- To be able to multiply and divide fractions involving algebraic expressions

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors, adding or subtracting with a lowest common denominator (LCD) and dividing, by multiplying by the reciprocal of the fraction that follows the division sign. In this section we focus on multiplying and dividing algebraic fractions.

The study of air-conditioning uses algebraic fractions to model airflow, air temperatures and humidity. The mechanical engineers who design ventilation systems, and the electricians who install and repair them, all require algebraic skills.

LESSON STARTER Describe the error

Here are three problems involving algebraic fractions. Each simplification contains one critical error. Find and describe the errors, then give the correct answer.

a $\frac{6x - \cancel{8}^{2}}{\cancel{4}_{1}} = \frac{6x - 2}{1}$
$= 6x - 2$

b $\frac{2a}{9} \div \frac{2}{3} = \frac{2a}{9} \times \frac{2}{3}$
$= \frac{4a}{27}$

c $\frac{3b}{7} \div \frac{2b}{3} = \frac{3b}{7} \times \frac{3b}{2}$
$= \frac{9b^2}{14}$

KEY IDEAS

- Simplify **algebraic fractions** by factorising expressions where possible and cancelling common factors.
- For multiplication, cancel common factors and then multiply the numerators together and the denominators together.
- For division, multiply by the **reciprocal** of the fraction that follows the division sign. The reciprocal of a is $\frac{1}{a}$ and the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

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BUILDING UNDERSTANDING

1 Simplify to find the answer in simplest form.

a $\frac{2}{3} \times \frac{6}{4}$ **b** $\frac{3}{4} \times \frac{10}{9}$ **c** $\frac{4}{7} \div \frac{2}{7}$ **d** $\frac{3}{6} \div \frac{6}{9}$

2 What is the reciprocal of each fraction?

a $\frac{3}{2}$ **b** $\frac{7a}{3}$ **c** $\frac{-4xy}{7t}$ **d** $\frac{-8x^2a}{b^2c}$

3 Simplify by cancelling common factors.

a $\frac{10x}{2}$ **b** $\frac{24x}{6}$ **c** $\frac{5a}{20}$ **d** $\frac{7}{21a}$

Example 6 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{8a^2b}{2a}$

b $\frac{3-9x}{3}$

SOLUTION

a $\frac{8a^2b}{2a} = \frac{\cancel{8}^4 \times \cancel{a}^1 \times a \times b}{\cancel{2}_1 \times \cancel{a}_1}$
$= 4ab$

b $\frac{3-9x}{3} = \frac{\cancel{3}^1(1-3x)}{\cancel{3}^1}$
$= 1 - 3x$

EXPLANATION

a Cancel the common factors 2 and a.

b Factorise the numerator, then cancel the common factor of 3.

Now you try

Simplify by cancelling common factors.

a $\frac{9ab^2}{3b}$

b $\frac{5-10x}{5}$

Example 7 Multiplying and dividing algebraic fractions

Simplify the following.

a $\frac{2}{a} \times \frac{a+2}{4}$

b $\frac{2x-4}{3} \div \frac{x-2}{6}$

SOLUTION

a $\frac{\cancel{2}^1}{a} \times \frac{a+2}{\cancel{4}_2} = \frac{a+2}{2a}$

EXPLANATION

a Cancel the common factor of 2 and then multiply the numerators and the denominators. a cannot be cancelled as it is not a common factor in $a + 2$.

ISBN 978-1-108-77346-1

Cambridge University Press

b $\frac{2x-4}{3} \div \frac{x-2}{6} = \frac{2x-4}{3} \times \frac{6}{x-2}$

$= \frac{2\cancel{(x-2)}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{6}^{2}}{\cancel{(x-2)}^{1}}$

$= 4$

Multiply by the reciprocal of the second fraction.

Factorise $2x - 4$ and cancel the common factors.

Now you try

Simplify the following.

a $\frac{6}{a} \times \frac{a+1}{12}$ **b** $\frac{3x-12}{2} \div \frac{x-4}{4}$

Exercise 1B

FLUENCY	1, 2–5(1/2)	2–5(1/2)	2–5(1/3)

1 Simplify by cancelling common factors.

Example 6a **a** **i** $\frac{6a^2b}{2a}$ **ii** $\frac{10xy^2}{5y}$

Example 6b **b** **i** $\frac{4-8x}{4}$ **ii** $\frac{5-5x}{5}$

Example 6a **2** Simplify by cancelling common factors.

a $\frac{35x^2}{7x}$ **b** $\frac{-14x^2y}{7xy}$ **c** $\frac{-36ab^2}{4ab}$ **d** $\frac{8xy^3}{-4xy^2}$

e $\frac{-15pq^2}{30p^2q^2}$ **f** $\frac{-20s}{45s^2t}$ **g** $\frac{-48x^2}{16xy}$ **h** $\frac{120ab^2}{140ab}$

Example 6b **3** Simplify by cancelling common factors.

a $\frac{4x+8}{4}$ **b** $\frac{6a-30}{6}$ **c** $\frac{6x-18}{2}$ **d** $\frac{5-15y}{5}$

e $\frac{-2-12b}{-2}$ **f** $\frac{21x-7}{-7}$ **g** $\frac{9t-27}{-9}$ **h** $\frac{44-11x}{-11}$

i $\frac{x^2+2x}{x}$ **j** $\frac{6x-4x^2}{2x}$ **k** $\frac{a^2-a}{a}$ **l** $\frac{7a+14a^2}{21a}$

Example 7a **4** Simplify the following.

a $\frac{3}{x} \times \frac{x-1}{6}$ **b** $\frac{x+4}{10} \times \frac{2}{x}$ **c** $\frac{-8a}{7} \times \frac{7}{2a}$

d $\frac{x+3}{9} \times \frac{4}{x+3}$ **e** $\frac{y-7}{y} \times \frac{5y}{y-7}$ **f** $\frac{10a^2}{a+6} \times \frac{a+6}{4a}$

g $\frac{2m+4}{m} \times \frac{m}{m+2}$ **h** $\frac{6-18x}{2} \times \frac{5}{1-3x}$ **i** $\frac{b-1}{10} \times \frac{-5}{b-1}$

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Example 7b

5 Simplify the following.

a $\dfrac{x}{5} \div \dfrac{x}{15}$ **b** $\dfrac{x+4}{2} \div \dfrac{x+4}{6}$ **c** $\dfrac{6x-12}{5} \div \dfrac{x-2}{3}$

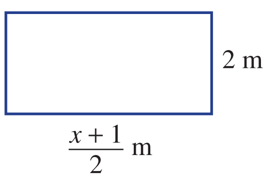
d $\dfrac{3-6y}{8} \div \dfrac{1-2y}{2}$ **e** $\dfrac{2}{a-1} \div \dfrac{3}{2a-2}$ **f** $\dfrac{2}{10x-5} \div \dfrac{10}{2x-1}$

g $\dfrac{5}{3a+4} \div \dfrac{15}{-15a-20}$ **h** $\dfrac{2x-6}{5x-20} \div \dfrac{x-3}{x-4}$ **i** $\dfrac{t+1}{9} \div \dfrac{-t-1}{3}$

PROBLEM-SOLVING	6	6, 7(1/2)	7–8(1/2)

6 Find a simplified expression for the area of these rectangles.

a



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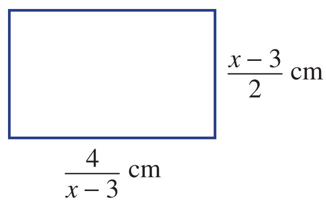
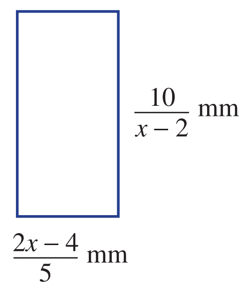


c



7 Simplify these expressions.

a $\dfrac{x}{3} \times \dfrac{9x}{5} \times \dfrac{15}{3x}$ **b** $\dfrac{2}{a} \times \dfrac{a}{5} \times \dfrac{10}{3a}$

c $\dfrac{x-1}{2} \times \dfrac{4x}{2x-2} \times \dfrac{x+3}{5x}$ **d** $\dfrac{2x-1}{x} \div \dfrac{2x-1}{2} \div \dfrac{1}{2}$

e $\dfrac{2x-3}{5} \div \dfrac{14x-21}{10} \div \dfrac{x}{2}$ **f** $\dfrac{b^2-b}{b} \div \dfrac{b-1}{b^2} \times \dfrac{2}{b-1}$

8 Write the missing algebraic fraction.

a $\dfrac{x+3}{5} \times \square = 2$ **b** $\dfrac{1-x}{x} \times \square = 3$

c $\square \div \dfrac{x}{2} = \dfrac{3(x+2)}{x}$ **d** $\square \div \dfrac{2x-2}{3} = \dfrac{5x}{x-1}$

e $\dfrac{1}{x} \div \square \times \dfrac{x-1}{2} = 1$ **f** $\dfrac{2-x}{7} \times \square \div \dfrac{5x}{x-1} = x$

REASONING	9(1/2)	9(1/2), 10	9(1/2), 10, 11

9 Recall that $(x-1)^2 = (x-1)(x-1)$. Use this idea to simplify the following.

a $\dfrac{(x-1)^2}{x-1}$ **b** $\dfrac{3(x+2)^2}{x+2}$ **c** $\dfrac{4(x-3)^2}{2(x-3)}$

d $\dfrac{4(x+2)}{(x+2)^2}$ **e** $\dfrac{-5(1-x)}{(1-x)^2}$ **f** $\dfrac{(2x-2)^2}{x-1}$

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10 Prove that the following all simplify to 1.

a $\frac{5x+5}{15} \times \frac{3}{x+1}$ **b** $\frac{3x-21}{2-x} \times \frac{4-2x}{6x-42}$ **c** $\frac{10-5x}{2x+6} \div \frac{20-10x}{4x+12}$

11 **a** Explain why $\frac{x-1}{2} \times \frac{4}{1-x} = \frac{x-1}{2} \times \frac{-4}{x-1}$.

b Use this idea to simplify these expressions.

i $\frac{2-a}{3} \times \frac{7}{a-2}$ **ii** $\frac{6x-3}{x} \div \frac{1-2x}{4}$ **iii** $\frac{18-x}{3x-1} \div \frac{2x-36}{7-21x}$

ENRICHMENT: Simplifying with quadratics	–	–	12(1/2)

12 You may recall that to factorise a monic quadratic of the form $x^2 + bx + c$ we look for factors of c which add to b. So for example: $x^2 - x - 6 = (x-3)(x+2)$.

So:

$$\frac{x^2-x-6}{6} \times \frac{3}{x-3} = \frac{{}^{1}\cancel{(x-3)}(x+2)}{\cancel{6}^{2}} \times \frac{\cancel{3}^{1}}{\cancel{(x-3)}_{1}}$$
$$= \frac{x+2}{2}$$

Now simplify these algebraic fractions which involve quadratics.

a $\frac{x^2-2x-8}{4} \times \frac{2}{x-4}$ **b** $\frac{x^2+5x+6}{x+2} \times \frac{x}{x+3}$

c $\frac{x+1}{x^2-4x-5} \times \frac{x-5}{3}$ **d** $\frac{3x-27}{4x} \times \frac{2x}{x^2-7x-18}$

e $\frac{4ab}{a^2+a} \div \frac{b}{a^2+2a+1}$ **f** $\frac{a+8}{a^2-5a-6} \div \frac{a^2+5a-24}{a-6}$

g $\frac{(x-y)^2}{xy} \div \frac{x^2-y^2}{x+y}$ **h** $\frac{y^2+4y+4}{x^2y} \div \frac{(y+2)^2}{xy^2+2xy}$

ISBN 978-1-108-77346-1

1C Adding and subtracting algebraic fractions

Learning intentions

- To know how to find the lowest common denominator of algebraic fractions
- To be able to combine numerators using expansion and addition of like terms
- To be able to add and subtract algebraic fractions

The sum or difference of two or more algebraic fractions can be simplified in a similar way to numerical fractions with the use of a common denominator.

Electricians, electrical and electronic engineers work with algebraic fractions when modelling the flow of electric energy in circuits. The application of algebra when using electrical formulas is essential in these professions.

LESSON STARTER Spot the difference

Here are two sets of simplification steps. One set has one critical error. Can you find and correct it?

$$\frac{2}{3}-\frac{5}{2}=\frac{4}{6}-\frac{15}{6}$$
$$=\frac{-11}{6}$$

$$\frac{x}{3}-\frac{x+1}{2}=\frac{2x}{6}-\frac{3(x+1)}{6}$$
$$=\frac{2x-3x+3}{6}$$
$$=\frac{-x+3}{6}$$

KEY IDEAS

- Add and subtract algebraic fractions by firstly finding the lowest common denominator (LCD) and then combine the numerators.
- Expand numerators correctly by taking into account addition and subtraction signs. E.g. $-2(x+1)=-2x-2$ and $-5(2x-3)=-10x+15$.

BUILDING UNDERSTANDING

1 Expand the following.

a $2(x-2)$ b $-(x+6)$ c $-6(x-2)$

2 Simplify these by firstly finding the lowest common denominator (LCD).

a $\frac{1}{2}+\frac{1}{3}$ b $\frac{4}{3}-\frac{1}{5}$ c $\frac{3}{7}-\frac{1}{14}$ d $\frac{5}{3}+\frac{7}{6}$

3 State the lowest common denominator for these pairs of fractions.

a $\frac{a}{3},\frac{7a}{4}$ b $\frac{x}{2},\frac{4xy}{6}$ c $\frac{3xy}{7},\frac{-3x}{14}$ d $\frac{2}{x},\frac{3}{2x}$

ISBN 978-1-108-77346-1

Example 8 Adding and subtracting simple algebraic fractions

Simplify the following.

a $\frac{3}{4}-\frac{a}{2}$

b $\frac{2}{5}+\frac{3}{a}$

SOLUTION

a $\frac{3}{4}-\frac{a}{2}=\frac{3}{4}-\frac{2a}{4}$
$=\frac{3-2a}{4}$

b $\frac{2}{5}+\frac{3}{a}=\frac{2a}{5a}+\frac{15}{5a}$
$=\frac{2a+15}{5a}$

EXPLANATION

The LCD of 2 and 4 is 4. Express each fraction as an equivalent fraction with a denominator of 4.
Subtract the numerators.

The LCD of 5 and a is $5a$.

Add the numerators.

Now you try

Simplify the following.

a $\frac{5}{6}-\frac{a}{3}$

b $\frac{3}{4}+\frac{2}{a}$

Example 9 Adding and subtracting more complex algebraic fractions

Simplify the following algebraic expressions.

a $\frac{x+3}{2}+\frac{x-2}{5}$

b $\frac{2x-1}{3}-\frac{x-1}{4}$

SOLUTION

a $\frac{x+3}{2}+\frac{x-2}{5}=\frac{5(x+3)}{10}+\frac{2(x-2)}{10}$
$=\frac{5(x+3)+2(x-2)}{10}$
$=\frac{5x+15+2x-4}{10}$
$=\frac{7x+11}{10}$

EXPLANATION

LCD is 10.

Use brackets to ensure you retain equivalent fractions.
Combine the numerators, then expand the brackets and simplify.

Continued on next page

ISBN 978-1-108-77346-1

b $\dfrac{2x-1}{3}-\dfrac{x-1}{4}=\dfrac{4(2x-1)}{12}-\dfrac{3(x-1)}{12}$ — Express each fraction with the LCD of 12.

$=\dfrac{4(2x-1)-3(x-1)}{12}$ — Combine the numerators.

$=\dfrac{8x-4-3x+3}{12}$ — Expand the brackets: $4(2x-1)=8x-4$ and $-3(x-1)=-3x+3$.

$=\dfrac{5x-1}{12}$ — Simplify by collecting like terms.

Now you try

Simplify the following algebraic expressions.

a $\dfrac{x+1}{3}+\dfrac{x-2}{2}$

b $\dfrac{3x-2}{2}-\dfrac{x-2}{5}$

Example 10 Adding and subtracting with algebraic denominators

Simplify the algebraic expression $\dfrac{3}{x-6}-\dfrac{2}{x+2}$.

SOLUTION

$$\frac{3}{x-6}-\frac{2}{x+2}=\frac{3(x+2)}{(x-6)(x+2)}-\frac{2(x-6)}{(x-6)(x+2)}$$

$$=\frac{3(x+2)-2(x-6)}{(x-6)(x+2)}$$

$$=\frac{3x+6-2x+12}{(x-6)(x+2)}$$

$$=\frac{x+18}{(x-6)(x+2)}$$

EXPLANATION

$(x-6)(x+2)$ is the lowest common multiple of $(x-6)$ and $(x+2)$.
Combine the numerators and then expand the brackets.

Recall that $-2\times(-6)=12$.

Collect like terms to simplify.

Now you try

Simplify the expression $\dfrac{4}{x-5}-\dfrac{3}{x+1}$.

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Exercise 1C

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/3)

1 Simplify the following.

Example 8a

a i $\frac{1}{4}-\frac{a}{2}$ ii $\frac{3}{10}-\frac{a}{5}$

Example 8b

b i $\frac{1}{3}+\frac{2}{a}$ ii $\frac{3}{7}+\frac{5}{a}$

Example 8a

2 Simplify the following.

a $\frac{2}{3}+\frac{a}{7}$ b $\frac{3}{8}+\frac{a}{2}$ c $\frac{3}{10}-\frac{3b}{2}$ d $\frac{2}{5}+\frac{4x}{15}$

e $\frac{1}{9}-\frac{2a}{3}$ f $\frac{a}{3}-\frac{a}{5}$ g $\frac{2x}{5}-\frac{x}{4}$ h $\frac{6b}{7}-\frac{b}{14}$

Example 8b

3 Simplify the following.

a $\frac{2}{3}+\frac{5}{a}$ b $\frac{3}{4}+\frac{2}{a}$ c $\frac{7}{9}-\frac{3}{a}$ d $\frac{4}{b}-\frac{3}{4}$

e $\frac{2}{7}-\frac{3}{2b}$ f $\frac{3}{2y}-\frac{7}{9}$ g $\frac{-4}{x}-\frac{2}{3}$ h $\frac{-9}{2x}-\frac{1}{3}$

Example 9a

4 Simplify the following algebraic expressions.

a $\frac{x+3}{4}+\frac{x+2}{5}$ b $\frac{x+2}{3}+\frac{x+1}{4}$ c $\frac{x-3}{4}+\frac{x+2}{2}$

d $\frac{x+4}{3}+\frac{x-3}{9}$ e $\frac{2x+1}{2}+\frac{x-2}{3}$ f $\frac{3x+1}{5}+\frac{2x+1}{10}$

g $\frac{x-2}{8}+\frac{2x+4}{12}$ h $\frac{5x+3}{10}+\frac{2x-2}{4}$ i $\frac{3-x}{14}+\frac{x-1}{7}$

Example 9b

5 Simplify these algebraic fractions.

a $\frac{2x+1}{3}-\frac{x-1}{2}$ b $\frac{3x-1}{3}-\frac{2x-3}{4}$ c $\frac{x+6}{5}-\frac{x-4}{3}$

d $\frac{x-3}{2}-\frac{2x+1}{7}$ e $\frac{7x+2}{7}-\frac{x+2}{3}$ f $\frac{10x-4}{3}-\frac{2x+1}{6}$

g $\frac{4-x}{6}-\frac{1-x}{5}$ h $\frac{1-3x}{5}-\frac{x+2}{3}$ i $\frac{6-5x}{2}-\frac{2-7x}{4}$

PROBLEM-SOLVING	6(1/2)	6(1/2)	6(1/2), 7

Example 10

6 Simplify the following algebraic expressions.

a $\frac{5}{x+1}+\frac{2}{x+4}$ b $\frac{4}{x-7}+\frac{3}{x+2}$ c $\frac{1}{x-3}+\frac{2}{x+5}$

d $\frac{3}{x+3}-\frac{2}{x-4}$ e $\frac{6}{2x-1}-\frac{3}{x-4}$ f $\frac{4}{x-5}+\frac{2}{3x-4}$

g $\frac{5}{2x-1}-\frac{6}{x+7}$ h $\frac{2}{x-3}-\frac{3}{3x+4}$ i $\frac{8}{3x-2}-\frac{3}{1-x}$

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7 **a** Write the LCD for these pairs of fractions.

i $\frac{3}{a}, \frac{2}{a^2}$ **ii** $\frac{7}{x^2}, \frac{3+x}{x}$

b Now simplify these expressions.

i $\frac{2}{a} - \frac{3}{a^2}$ **ii** $\frac{a+1}{a} - \frac{4}{a^2}$ **iii** $\frac{7}{2x^2} + \frac{3}{4x}$

REASONING	8	8, 9	8, 9

8 Describe the error in this working, then fix the solution.

$$\begin{aligned}\frac{x}{2} - \frac{x+1}{3} &= \frac{3x}{6} - \frac{2(x+1)}{6}\\ &= \frac{3x}{6} - \frac{2x+2}{6}\\ &= \frac{x+2}{6}\end{aligned}$$

9 **a** Explain why $2x - 3 = -(3 - 2x)$.

b Use this idea to help simplify these expressions.

i $\frac{1}{x-1} - \frac{1}{1-x}$ **ii** $\frac{3x}{3-x} + \frac{x}{x-3}$ **iii** $\frac{x+1}{7-x} - \frac{2}{x-7}$

ENRICHMENT: Fraction challenges	–	–	10, 11

10 Simplify these expressions.

a $\frac{a-b}{b-a}$

b $\frac{5}{a} + \frac{2}{a^2}$

c $\frac{3}{x+1} + \frac{2}{(x+1)^2}$

d $\frac{x}{(x-2)^2} - \frac{x}{x-2}$

e $\frac{x}{2(3-x)} - \frac{x^2}{7(x-3)^2}$

f $\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$

11 By first simplifying the left-hand side of these equations, find the value of a.

a $\frac{a}{x-1} - \frac{2}{x+1} = \frac{4}{(x-1)(x+1)}$

b $\frac{3}{2x-1} + \frac{a}{x+1} = \frac{5x+2}{(2x-1)(x+1)}$

ISBN 978-1-108-77346-1

1D Solving linear equations

Learning intentions

- To know the form of a linear equation
- To understand that an equivalent equation can be generated by applying the same operation to each side of the equation
- To be able to solve a linear equation involving two or more steps, including brackets and variables on both sides
- To be able to solve linear equations involving algebraic fractions
- To understand that solutions can be checked by substituting into both sides of an equation

A linear equation is a statement that contains an equals sign and includes constants and pronumerals with a power of 1 only. Here are some examples:

$$2x - 5 = 7$$
$$\frac{x+1}{3} = x + 4$$
$$-3(x+2) = \frac{1}{2}$$

We solve linear equations by operating on both sides of the equation until a solution is found.

A small business, such as a garden nursery, generates revenue from its sales. To calculate the number of employees (x) a business can afford, a linear revenue equation is solved for x:
Revenue (y) = pay (m) × employees (x) + costs (c)

LESSON STARTER What's the best method?

Here are four linear equations.

- Discuss what you think is the best method to solve them using 'by hand' techniques.
- Discuss how it might be possible to check that a solution is correct.

a $\frac{7x-2}{3} = 4$ **b** $3(x-1) = 6$ **c** $4x + 1 = x - 2$ **d** $\frac{2x+1}{5} = \frac{x-1}{3}$

KEY IDEAS

- An equation is true for the given values of the pronumerals when the left-hand side equals the right-hand side.
  $2x - 4 = 6$ is true when $x = 5$ but false when $x \neq 5$.
- A **linear equation** contains pronumerals with a highest power of 1.
- Useful steps in solving linear equations are:
  - using inverse operations (backtracking)
  - collecting like terms
  - expanding brackets
  - multiplying by the lowest common denominator.

ISBN 978-1-108-77346-1

BUILDING UNDERSTANDING

1. Decide whether the following are linear equations.
   a $x^2 - 1 = 0$
   b $\sqrt{x} + x = 3$
   c $\frac{x-1}{2} = 5$
   d $\frac{3x}{4} = 2x - 1$
2. Decide whether these equations are true when $x = 2$.
   a $3x - 1 = 5$
   b $4 - x = 1$
   c $\frac{2x+1}{5} = x + 4$
3. Decide whether these equations are true when $x = -6$.
   a $-3x + 17 = x$
   b $2(4 - x) = 20$
   c $\frac{2-3x}{10} = \frac{-12}{x}$
4. Solve the following equations and check your solution using substitution.
   a $x + 8 = 13$
   b $x - 5 = 3$
   c $-x + 4 = 7$
   d $-x - 5 = -9$

Example 11 Solving linear equations

Solve the following equations and check your solution using substitution.

a $4x + 5 = 17$

b $3(2x + 5) = 4x$

SOLUTION

a
$$\begin{aligned} 4x + 5 &= 17 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

Check: LHS $= 4 \times 3 + 5 = 17$, RHS $= 17$.

b
$$\begin{aligned} 3(2x + 5) &= 4x \\ 6x + 15 &= 4x \\ 2x + 15 &= 0 \\ 2x &= -15 \\ x &= -\frac{15}{2} \end{aligned}$$

Check:

$$\begin{aligned} \text{LHS} &= 3\left(2 \times \left(-\frac{15}{2}\right) + 5\right) \\ &= -30 \\ \text{RHS} &= 4 \times \left(-\frac{15}{2}\right) \\ &= -30 \end{aligned}$$

EXPLANATION

a Subtract 5 from both sides and then divide both sides by 4.

Check by seeing if $x = 3$ makes the equation true.

b Expand the brackets.

Gather like terms by subtracting $4x$ from both sides.

Subtract 15 from both sides and then divide both sides by 2.

Check by seeing if $x = -\frac{15}{2}$ makes the equation true by substituting into the equation's left-hand side (LHS) and right-hand side (RHS) and confirming they are equal.

Now you try

Solve the following equations and check your solution using substitution.

a $2x + 7 = 13$

b $4(2x + 1) = 2x$

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Example 12 Solving equations involving algebraic fractions

Solve the following equations and check your solution using substitution.

a $\frac{x+3}{4} = 2$ b $\frac{4x-2}{3} = \frac{3x-1}{2}$ c $\frac{x+2}{3} - \frac{2x-1}{2} = 4$

SOLUTION

a
$$\frac{x+3}{4} = 2$$
Multiply both sides by 4.
$$x + 3 = 8$$
Subtract 3 from both sides.
$$\therefore x = 5$$
Check: LHS $= \frac{5+3}{4} = 2$, RHS $= 2$.

Check by seeing if $x = 5$ makes the equation true.

b
$$\frac{4x-2}{3} = \frac{3x-1}{2}$$
Multiply both sides by the LCD of 2 and 3, which is 6.
$$\frac{\cancel{6}^{2}(4x-2)}{\cancel{3}_{1}} = \frac{\cancel{6}^{3}(3x-1)}{\cancel{2}_{1}}$$
Cancel common factors.
$$2(4x-2) = 3(3x-1)$$
$$8x - 4 = 9x - 3$$
$$-4 = x - 3$$
$$-1 = x$$
$$\therefore x = -1$$
Expand the brackets and gather terms containing x by subtracting $8x$ from both sides. Rewrite with x as the subject. Check by seeing if $x = -1$ makes the equation true.

c
$$\frac{x+2}{3} - \frac{2x-1}{2} = 4$$
Multiply both sides by the LCD of 2 and 3, which is 6.
$$\frac{6(x+2)}{3} - \frac{6(2x-1)}{2} = 4 \times 6$$
$$2(x+2) - 3(2x-1) = 24$$
Cancel common factors.
$$2x + 4 - 6x + 3 = 24$$
Expand, noting that $-3 \times (-1) = 3$.
$$-4x + 7 = 24$$
Simplify and solve for x.
$$-4x = 17$$
$$\therefore x = -\frac{17}{4}$$
Check your solution using substitution.

Now you try

Solve the following equations and check your solution using substitution.

a $\frac{x+1}{3} = 4$ b $\frac{2x-1}{3} = \frac{x-3}{4}$ c $\frac{x+1}{2} - \frac{2x-1}{3} = 2$

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Exercise 1D

FLUENCY	1, 2–5(1/2)	2–5(1/3)	2–5(1/4)

Example 11a
Example 11b

1 Solve the following equations and check your solution using substitution.

a i $3x + 4 = 13$ ii $5x + 2 = 27$

b i $2(2x + 3) = 2x$ ii $5(x + 2) = 3x$

Example 11a

2 Solve the following equations and check your solution using substitution.

a $2x + 9 = 14$ b $4x + 3 = 14$ c $3x - 3 = -4$ d $6x + 5 = -6$

e $-3x + 5 = 17$ f $-2x + 7 = 4$ g $-4x - 9 = 9$ h $-3x - 7 = -3$

i $8 - x = 10$ j $5 - x = -2$ k $6 - 5x = 16$ l $4 - 9x = -7$

Example 11b

3 Solve the following equations and check your solution using substitution.

a $4(x + 3) = 16$ b $2(x - 3) = 12$ c $2(x - 4) = 15$

d $3(1 - 2x) = 8$ e $3(2x + 3) = -5x$ f $2(4x - 5) = -7x$

g $3(2x + 3) + 2(x + 4) = 25$ h $2(2x - 3) + 3(4x - 1) = 23$ i $2(3x - 2) - 3(x + 1) = 5$

j $5(2x + 1) - 3(x - 3) = 63$ k $5(x - 3) = 4(x - 6)$ l $4(2x + 5) = 3(x + 15)$

m $5(x + 2) = 3(2x - 3)$ n $3(4x - 1) = 7(2x - 7)$ o $7(2 - x) = 8 - x$

Example 12a

4 Solve the following equations and check your solution using substitution.

a $\frac{x - 4}{2} = 3$ b $\frac{x + 2}{3} = 5$ c $\frac{x + 4}{3} = -6$ d $\frac{2x + 7}{3} = 5$

e $\frac{2x + 1}{3} = -3$ f $\frac{3x - 2}{4} = 4$ g $\frac{x}{2} - 5 = 3$ h $\frac{3x}{2} + 2 = 8$

i $\frac{2x}{3} - 2 = -8$ j $5 - \frac{x}{2} = 1$ k $4 - \frac{2x}{3} = 0$ l $5 - \frac{4x}{7} = 9$

m $\frac{x + 1}{3} + 2 = 9$ n $\frac{x - 3}{2} - 4 = 2$ o $4 + \frac{x - 5}{2} = -3$ p $1 - \frac{2 - x}{3} = 2$

5 For each of the following statements, write an equation and solve it to find x.

a When 3 is added to x, the result is 7.

b When x is added to 8, the result is 5.

c When 4 is subtracted from x, the result is 5.

d When x is subtracted from 15, the result is 22.

e Twice the value of x is added to 5 and the result is 13.

f 5 less than x when doubled is -15.

g When 8 is added to 3 times x, the result is 23.

h 5 less than twice x is 3 less than x.

PROBLEM-SOLVING	7, 8, 11	6(1/2), 7, 9, 11, 13(1/2)	6–7(1/2), 9–12, 13(1/2)

Example 12b

6 Solve the following equations, which involve algebraic fractions.

a $\frac{2x + 12}{7} = \frac{3x + 5}{4}$ b $\frac{5x - 4}{4} = \frac{x - 5}{5}$ c $\frac{3x - 5}{4} = \frac{2x - 8}{3}$

d $\frac{1 - x}{5} = \frac{2 - x}{3}$ e $\frac{6 - 2x}{3} = \frac{5x - 1}{4}$ f $\frac{10 - x}{2} = \frac{x + 1}{3}$

g $\frac{2(x + 1)}{3} = \frac{3(2x - 1)}{2}$ h $\frac{-2(x - 1)}{3} = \frac{2 - x}{4}$ i $\frac{3(6 - x)}{2} = \frac{2(x + 1)}{5}$

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7 Substitute the given values and then solve for the unknown in each of the following common formulas.

a $v = u + at$ Solve for a given $v = 6$, $u = 2$ and $t = 4$.

b $s = ut + \frac{1}{2}at^2$ Solve for u given $s = 20$, $t = 2$ and $a = 4$.

c $A = h\left(\frac{a + b}{2}\right)$ Solve for b given $A = 10$, $h = 4$ and $a = 3$.

d $A = P\left(1 + \frac{r}{100}\right)$ Solve for r given $A = 1000$ and $P = 800$.

8 The perimeter of a square is 68 cm. Determine its side length.

9 The sum of two consecutive numbers is 35. What are the numbers?

10 I ride four times faster than I jog. If a trip took me 45 minutes and I spent 15 of these minutes jogging 3 km, how far did I ride?

11 A service technician charges \$30 up front and \$46 for each hour that he works.

a What will a 4-hour job cost?

b If the technician works on a job for 2 days and averages 6 hours per day, what will be the overall cost?

c Find how many hours the technician worked if the cost is:

i \$76

ii \$513

iii \$1000 (round to the nearest half hour).

12 The capacity of a petrol tank is 80 litres. If it initially contains 5 litres and a petrol pump fills it at 3 litres per 10 seconds, find:

a the amount of fuel in the tank after 2 minutes

b how long it will take to fill the tank to 32 litres

c how long it will take to fill the tank.

Example 12c

13 Solve the following equations by multiplying both sides by the LCD.

a $\frac{x}{2} + \frac{2x}{3} = 7$

b $\frac{x}{4} + \frac{3x}{3} = 5$

c $\frac{3x}{5} - \frac{2x}{3} = 1$

d $\frac{2x}{5} - \frac{x}{4} = 3$

e $\frac{x - 1}{2} + \frac{x + 2}{5} = 2$

f $\frac{x + 3}{3} + \frac{x - 4}{2} = 4$

g $\frac{x + 2}{3} - \frac{x - 1}{2} = 1$

h $\frac{x - 4}{5} - \frac{x + 2}{3} = 2$

i $\frac{7 - 2x}{3} - \frac{6 - x}{2} = 1$

ISBN 978-1-108-77346-1

Cambridge University Press

REASONING	14	14	15

14 Solve $2(x - 5) = 8$ using the following two methods and then decide which method you prefer. Give a reason.

a Method 1: First expand the brackets.

b Method 2: First divide both sides by 2.

15 A family of equations can be represented using other pronumerals (sometimes called parameters). For example, the solution to the family of equations $2x - a = 4$ is $x = \dfrac{4 + a}{2}$.

Find the solution for x in these equation families.

a $x + a = 5$
b $6x + 2a = 3a$
c $ax + 2 = 7$
d $ax - 1 = 2a$
e $\dfrac{ax - 1}{3} = a$
f $ax + b = c$

ENRICHMENT: More complex equations	–	–	16, 17

16 Make a the subject in these equations.

a $a(b + 1) = c$
b $ab + a = b$
c $\dfrac{1}{a} + b = c$
d $a - \dfrac{a}{b} = 1$
e $\dfrac{1}{a} + \dfrac{1}{b} = 0$
f $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{c}$

17 Solve for x in terms of the other pronumerals.

a $\dfrac{x}{2} - \dfrac{x}{3} = a$
b $\dfrac{x}{a} + \dfrac{x}{b} = 1$
c $\dfrac{x}{a} - \dfrac{x}{b} = c$

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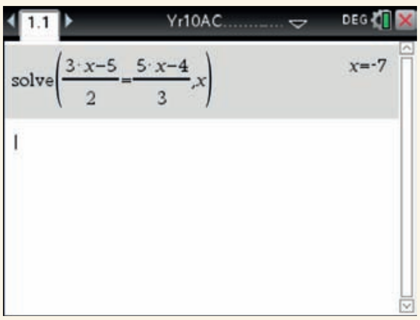
Using calculators to solve equations and inequalities (CAS)

1 Solve the equation $\dfrac{3x - 5}{2} = \dfrac{5x - 4}{3}$.

2 Solve the inequality $5 < \dfrac{2x + 3}{5}$.

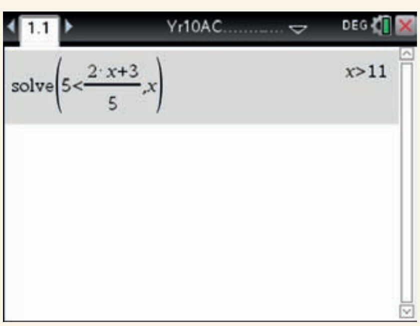
Using the TI-Nspire:

1 In a **Calculator** page use menu **>Algebra>Solve** and type the equation as shown.



Hint: use the fraction template (ctrl ÷)

2 In a **Calculator** page use menu **>Algebra>Solve** and type the equation as shown.

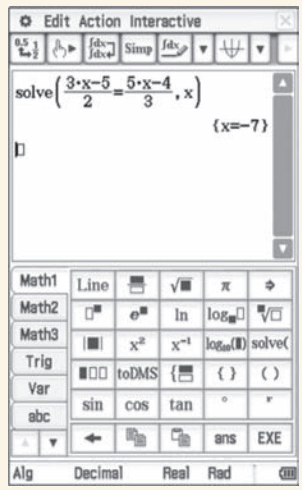


Hint: the inequality symbols (e.g. <) are accessed using ctrl =

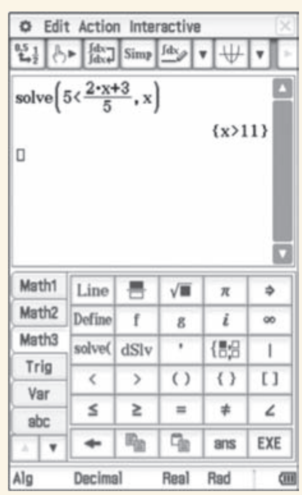
Hint: use the fraction template (ctrl ÷)

Using the ClassPad:

1 Tap **solv(**, then and type the equation as shown.



2 Tap **solv(** and type the equation as shown.



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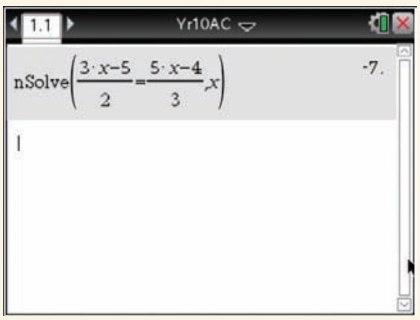
Using calculators to solve equations (non-CAS)

1 Solve the equation $\dfrac{3x-5}{2} = \dfrac{5x-4}{3}$.

2 Solve the equation $\dfrac{x-1}{4} = \dfrac{3x+4}{3}$.

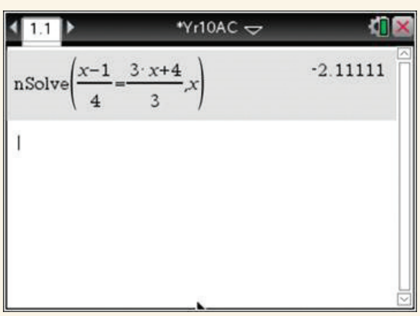
Using the TI-Nspire CX non-CAS:

1 In a **Calculator** page use menu **>Algebra>Numerical Solve** and type the equation as shown.



Hint: use the fraction template (ctrl ÷)

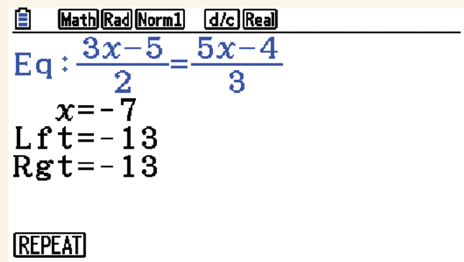
2 In a **Calculator** page use menu **>Algebra>Numerical Solve** and type the equation as shown.



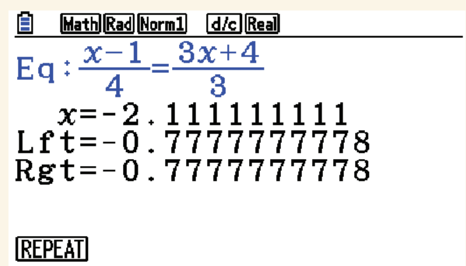
An approximate answer is given to this equation.
Hint: use the fraction template (ctrl ÷)

Using the Casio:

1 In the **Equation** application, select **F3: Solver**. Enter the equation as shown and press **EXE** and select **F6 (SOLVE)** to obtain the solution.



2 In the **Equation** application, select **F3: Solver**. Enter the equation as shown and press **EXE** and select **F6 (SOLVE)** to obtain the solution.



ISBN 978-1-108-77346-1

1E Linear inequalities

Learning intentions

- To know the meaning of the term inequality
- To be able to use and interpret the signs $>, \geqslant, \leqslant, <$
- To know how to interpret and represent an inequality on a number line
- To understand when to reverse the sign in an inequality
- To be able to solve a linear inequality

There are many situations in which a solution to the problem is best described using one of the symbols $<, \leqslant, >$ or $\geqslant$. For example, a pharmaceutical company might need to determine the possible number of packets of a particular drug that need to be sold so that the product is financially viable. This range of values may be expressed using inequality symbols.

Doctors, nurses and pharmacists can use an inequality to express the dosage range of a medication from the lowest effective level to the highest safe level.

An inequality is a mathematical statement that uses one of the following symbols: *is less than* ($<$), is *less than or equal to* ($\leqslant$), *is greater than* ($>$) *or is greater than or equal to* $\geqslant$. Inequalities may result in an infinite number of solutions and these can be illustrated using a number line.

LESSON STARTER What does it mean for x?

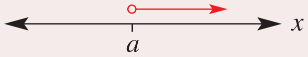
The following inequalities provide some information about the value of x.

a $2 \geqslant x$ **b** $-2x < 4$ **c** $3 - x \leqslant -1$

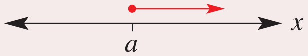
- Can you describe the possible values for x that satisfy each inequality?
- Test some values to check.
- How would you write the solution for x? Illustrate this on a number line.

KEY IDEAS

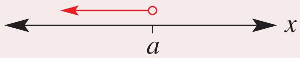
- The four **inequality signs** are $<, \leqslant, >$ and $\geqslant$.
  - $x > a$ means x is greater than a.



  - $x \geqslant a$ means x is greater than or equal to a.



  - $x < a$ means x is less than a.



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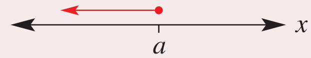
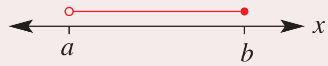
- $x \leqslant a$ means x is less than or equal to a.



- Also $a < x \leqslant b$ could be illustrated as shown.



- An open circle is used for $<$ or $>$ where the end point is not included.
- A closed circle is used for $\leqslant$ or $\geqslant$ where the end point is included.

■ Solving **linear inequalities** follows the same rules as solving linear equations, except:
- We reverse the inequality sign if we multiply or divide by a negative number.
  For example, $-5 < -3$ is equivalent to $5 > 3$ and if $-2x < 4$, then $x > -2$.
- We reverse the inequality sign if the sides are switched.
  For example, if $2 \geqslant x$, then $x \leqslant 2$.

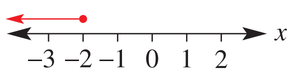
BUILDING UNDERSTANDING

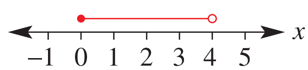
1 State three numbers that satisfy each of these inequalities.

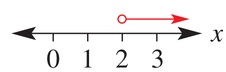
a $x \geqslant 3$ **b** $x < -1.5$ **c** $0 < x \leqslant 7$ **d** $-8.7 \leqslant x < -8.1$

2 Match the graph **a–c** with the inequality **A–C**.

A $x > 2$ **B** $x \leqslant -2$ **C** $0 \leqslant x < 4$

a


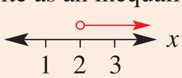
b


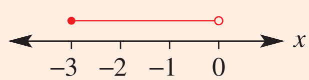
c


3 Phil houses x rabbits. If $10 < x \leqslant 13$, how many rabbits could Phil have?

Example 13 Writing inequalities from number lines

Write as an inequality.

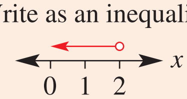
a


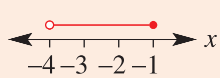
b


SOLUTION	EXPLANATION
a $x > 2$	An open circle means 2 is not included.
b $-3 \leqslant x < 0$	-3 is included but 0 is not.

Now you try

Write as an inequality.

a


b


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